Period 4, Feb 4, 2025

The Law of Sines

If A, B, and C are the measures of the angles of a triangle, and a, b, and c are the lengths of the sides opposite these angles, then

$$\left(\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}\right) \quad \left(\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}\right)$$

The ratio of the length of the side of any triangle to the sine of the angle opposite that side is the same for all three sides of the triangle.

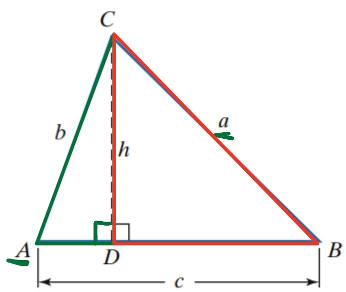


Figure 6.2 Drawing an altitude to prove the Law of Sines

$$b \cdot \sin A = \frac{h}{\cancel{b}} \cdot b$$

$$a \cdot \sin B = \frac{h}{\cancel{a}} \cdot a$$

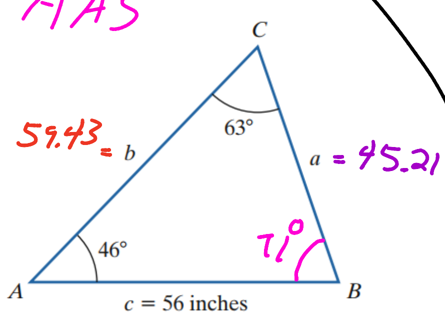
$$b \sin A = h$$

$$a \sin B = h$$

$$\frac{b \sin A}{\cancel{\sin A} \sin B} = \frac{a \sin B}{\sin A \cancel{\sin B}}$$

$$\frac{b}{\sin B} = \frac{a}{\sin A}$$

AAS



$$46 + 63 + B = 180$$

$$\cancel{109} + B = 180$$
$$-10\cancel{9} \qquad -109$$

$$B = 71$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{\sin 46} = \frac{b}{\sin 71} = \frac{56}{\sin 63} = \frac{56}{0.8910065}$$

$$\cancel{\sin 46} \cdot \frac{a}{\cancel{\sin 46}} = 62.85027 \cdot \sin 46$$

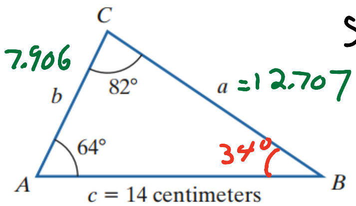
$$a = 62.85027 \cdot (.7193398) = 45.21$$

$$\frac{b}{\sin 71} = \frac{56}{\sin 63} \Rightarrow \cancel{\sin 71}\,\frac{b}{\cancel{\sin 71}} = 62.85027 \cdot \sin 71$$

$$b = (62.85027)(.9455186)$$

$$b = 59.43$$

AAS



$$\frac{a}{\sin 64} = \frac{b}{\sin B} = \frac{14}{\sin 82}$$

$$\frac{a}{\sin 64} = \frac{b}{\sin 34} = \frac{14}{\sin 82}$$

$$\frac{14}{\sin 82} = \frac{14}{0.99026807} = 14.13759$$

$$A + B + C = 180^\circ$$

$$64^\circ + B + 82^\circ = 180^\circ$$

$$B = 180 - 146 = 34$$

$$\cancel{\sin 64} \cdot \frac{a}{\cancel{\sin 64}} = 14.13759 \cdot \sin 64$$

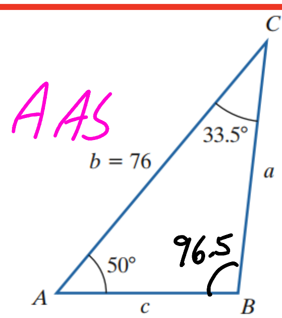
$$a = 14.13759 \cdot (.898794046)$$

$$a = 12.707$$

$$\cancel{\sin 34}\,\frac{b}{\cancel{\sin 34}} = 14.13759 \cdot \sin 34$$

$$b = 14.13759 \cdot (0.559192903) = 7.906$$

AAS



$a = 58.596$

$c = 42.219$

$B = 96.5^\circ$

$$\frac{a}{\sin 50} = \frac{76}{\sin 96.5} = \frac{c}{\sin 33.5}$$

$$\frac{a}{\sin 50} = \frac{76}{.9935719} = \frac{c}{\sin 33.5}$$

$$50 + 33.5 + B = 180$$

$$83.5 + B = 180$$

$$B = 180 - 83.5$$

$$= 96.5$$

$$\cancel{\sin 50}\,\frac{a}{\cancel{\sin 50}} = 76.4916997 \cdot \sin 50$$

$$a = 58.596$$

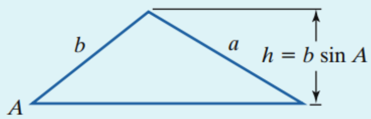
$$\cancel{\sin 33.5}\,\frac{c}{\cancel{\sin 33.5}} = 76.4916997 \cdot \sin 33.5$$

$$c = 42.2186$$

The Ambiguous Case (SSA)

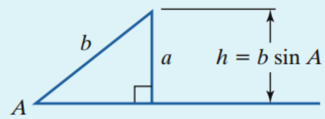
Consider a triangle in which a, b, and A are given. This information may result in

One Triangle



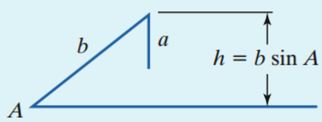
a is greater than h and a is greater than b. One triangle is formed.

One Right Triangle



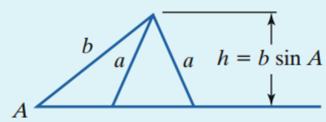
$a = h$ and is just the right length to form a right triangle.

No Triangle



a is less than h and is not long enough to form a triangle.

Two Triangles

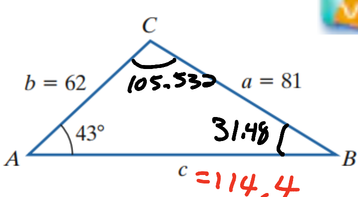


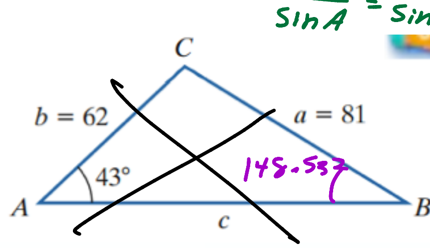
a is greater than h and a is less than b. Two distinct triangles are formed.

$C + 43 + 31.468 = 180$

SSA $C = 105.532$

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$





$\frac{81}{\sin 43} = \frac{62}{\sin B} = \frac{c}{\sin C} = \frac{c}{\sin 105.532}$

$\sin b \cdot \frac{81}{0.681998} = \frac{62}{\cancel{\sin b}} \cdot \cancel{\sin b}$

$\frac{(\sin b) \cdot (\cancel{118.768614})}{\cancel{118.768614}} = \frac{62}{118.768614}$

$\sin B = 0.522023$

$B = \sin^{-1} 0.522023$

$B = 31.468$

$\sin 31.468 = \sin 148.532$

$\frac{81}{\sin 43} = \frac{62}{\sin B} = \frac{c}{\sin C}$

$B = 148.532$

$43 + 148.532 + C = 180$

$191.532 + C = 180$

$C = -11.532^\circ$

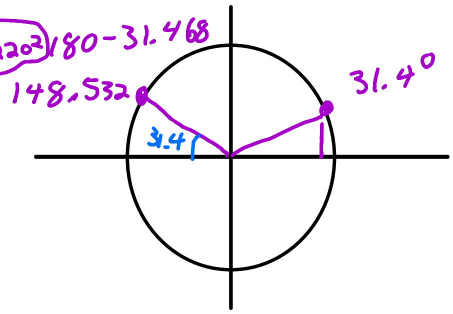
Not Possible

$118.768614 = \frac{c}{\sin 105.532} = \frac{c}{.963481}$

$(118.768614)(.963481) = c$

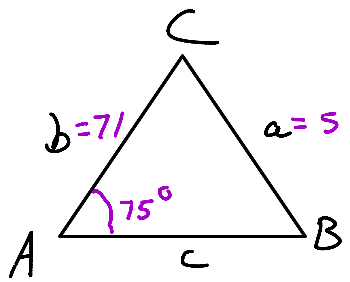
$114.4 = c$

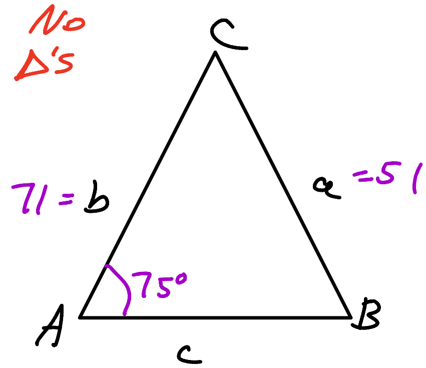
$\sin 148.532 = 0.52202$



Solve triangle ABC if $A = 75°$, $a = 51$, and $b = 71$.

No Δ's





$$\frac{51}{\sin 75} = \frac{71}{\sin B}$$

$$\sin B \cdot 52.799085 = \frac{71}{\sin B} \cdot \sin B$$

$$52.799085 \sin B = 71$$

$$\sin B = \frac{71}{52.799085} = 1.3447$$

$$\sin^{-1} 1.3447 = \emptyset$$

$$\sin B \leq 1 \qquad \cancel{\sin B > 1}$$

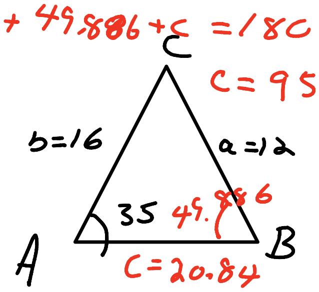
CHECK POINT 5 Solve triangle ABC if $A = 35°$, $a = 12$, and $b = 16$.

$$\frac{12}{\sin 35} = \frac{16}{\sin B} = \frac{c}{\sin C}$$

$$\frac{12}{\sin 35} = \frac{12}{0.573576} = 20.921362$$

$35 + 49.886 + C = 180$

$C = 95.114$



$\frac{12}{\sin 35} = \frac{16}{\sin B}$

$\sin B \cdot 20.9213 = \frac{16}{\cancel{\sin B}} \cdot \cancel{\sin B}$

$20.921362 \cdot \sin B = 16$

$\sin B = \frac{16}{20.921362}$

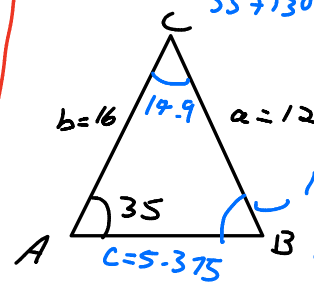
$\sin B = .7647686$

$B = \sin^{-1} .7647686$

$B = 49.886$

$35 + 130.114 + C = 180$

$\boxed{C = 14.886}$



$180 - 49.886$

$B = 130.114$

$\frac{12}{\sin 35} = \frac{c}{\sin 14.886}$

$.256896 \cdot 20.921362 = \frac{c}{\cancel{.2568967}} \cdot \cancel{.2568967}$

0.2568967

$5.375 = c$

$\frac{a}{\sin a} = \frac{c}{\sin c}$

$\frac{12}{\sin 35} = \frac{c}{\sin 95.114}$

$.996014 \cdot 20.921362 = \frac{c}{.996019} \cdot .996019$

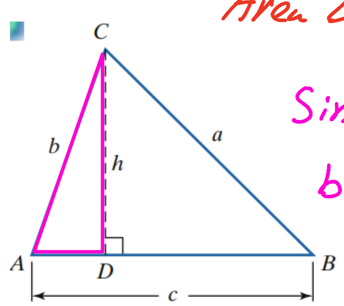
$\Rightarrow c = 20.84$

Area of an Oblique SAS Triangle

The area of a triangle equals one-half the product of the lengths of two sides times the sine of their included angle. In **Figure 6.10**, this wording can be expressed by the formulas

$$\text{Area} = \boxed{\tfrac{1}{2}bc\sin A} = \tfrac{1}{2}ab\sin C = \tfrac{1}{2}ac\sin B.$$

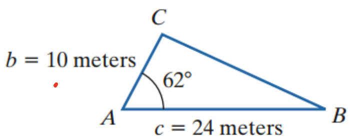
$\text{Area } \triangle = \frac{1}{2}(\text{base} \cdot \text{height}) = \frac{1}{2} \cdot c \cdot b \cdot \sin A$



$\sin A = \frac{h}{b}$

$b \cdot \sin A = h$

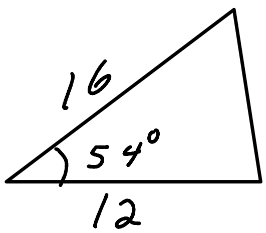
You need 2 sides and The angle between Them



$A = \frac{1}{2}(24)(10) \cdot \sin 62$

$A = 120 \cdot (.8829476)$

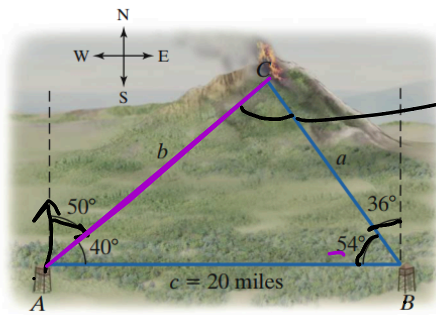
$Area = 105.954\ m^2$



$Area = \frac{1}{2}(16)(12) \sin 54$

$77.67 = 96 \cdot (.809016994)$

Two fire-lookout stations are 20 miles apart, with station B directly east of station A. Both stations spot a fire on a mountain to the north. The bearing from station A to the fire is N50°E (50° east of north). The bearing from station B to the fire is N36°W (36° west of north). How far, to the nearest tenth of a mile, is the fire from station A?



$C + 40 + 54 = 180$

$C = 86°$

$$\frac{20}{\sin 86} = \frac{b}{\sin 54}$$

$$(0.809016994)\ 20.04884 = \frac{b}{\cancel{0.80901694}}\ \cancel{(0.80901699)}$$

$$16.219 = b$$

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$
$$\sin\alpha - \sin\beta = 2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}$$
$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$
$$\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

Product-to-Sum Formulas

$$\sin\alpha\sin\beta = \tfrac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$$
$$\cos\alpha\cos\beta = \tfrac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)]$$
$$\sin\alpha\cos\beta = \tfrac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$$
$$\cos\alpha\sin\beta = \tfrac{1}{2}[\sin(\alpha+\beta) - \sin(\alpha-\beta)]$$

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$
$$\sin\alpha - \sin\beta = 2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}$$

Double-Angle Formulas

$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$
$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

Power-Reducing Formulas

$$\sin^2\theta = \frac{1-\cos 2\theta}{2} \qquad \cos^2\theta = \frac{1+\cos 2\theta}{2} \qquad \tan^2\theta = \frac{1-\cos 2\theta}{1+\cos 2\theta}$$

$$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$$
$$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$$
$$\tan\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$$

9. $\sin\theta \tan\theta + \cos\theta = \dfrac{1}{\cos\theta}$

$$\frac{\sin\theta}{1}\cdot\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{1}$$

$$\frac{\sin^2\theta}{\cos\theta} + \frac{\cos\theta\cdot\cos\theta}{1\cdot\cos\theta}$$

$$\frac{\sin^2\theta}{\cos\theta} + \frac{\cos^2\theta}{\cos\theta}$$

$$\frac{\sin^2\theta+\cos^2\theta}{\cos\theta} = \frac{1}{\cos\theta}$$

$$\theta \neq \frac{(2k+1)\pi}{2}$$

$\cos\theta \neq 0$

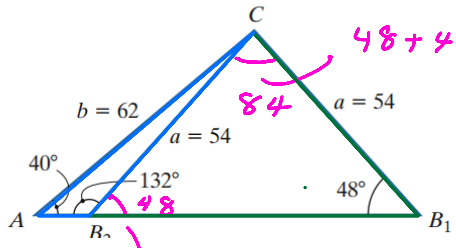
$$\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \ldots$$

$$\frac{2\pi}{2} \quad +\frac{2\pi}{2}$$

$$\frac{2\pi}{2}\cdot k + \frac{\pi}{2}$$

$$\frac{2\pi k + \pi}{2}$$

$$\theta \neq \frac{\pi(2k+1)}{2}$$

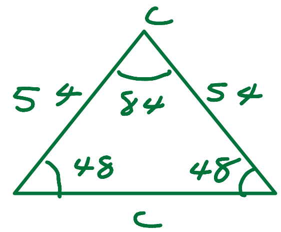


$48+48+c=180$

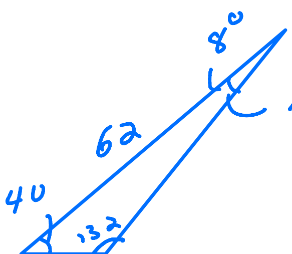
$96+c=180$

84

$180-132$



$$\frac{54}{\sin 48}=\frac{c}{\sin 84}$$



$180=40+132+c$

$8^\circ=c$

$c=11.75$

$$\frac{62}{\sin 132}=\frac{c}{\sin 8}$$

$$84.429=\frac{c}{0.13917}$$

$(0.13917)(84.429)=c$

$11.75=c$

$\tan^2\alpha+1=\sec^2\alpha$

$\left(-\frac{5}{12}\right)^2+1=\sec^2\alpha$

$$\frac{25}{144}+\frac{144}{144}=\sqrt{\frac{169}{144}}=\sqrt{\sec^2\alpha}$$

$$\pm\frac{13}{12}=\frac{1}{\cos\alpha}$$

$$\pm\frac{12}{13}=\cos\alpha$$

Find the exact value of each of the following under the given conditions.

$(-,+)$ $(+,+)$

tan $\alpha=-\frac{5}{12}$, α lies in quadrant II, and **cos** $\beta=\frac{7}{8}$, β lies in quadrant I

a. sin $(\alpha+\beta)$ **b. cos** $(\alpha+\beta)$ **c. tan** $(\alpha+\beta)$

$$\rightarrow -\frac{12}{13}=\cos\alpha$$

$\sin^2\beta+\cos^2\beta=1$

$$\sin^2\beta+\left(\frac{7}{8}\right)^2=\frac{64}{64}$$

$$\sin^2\beta+\frac{49}{64}=\frac{64}{64}$$

$$\sqrt{\sin^2\beta}=\sqrt{\frac{15}{64}}$$

$$\sin\beta=\frac{+\sqrt{15}}{8}$$

$$\sin\alpha=+\frac{5}{13}$$

$\sin^2\alpha+\cos\alpha=1$

$$\sin^2\alpha+\left(-\frac{12}{13}\right)^2=\frac{169}{169}$$

$$\sin^2\alpha+\cancel{\frac{144}{169}}=\frac{169}{169}-\frac{144}{169}$$

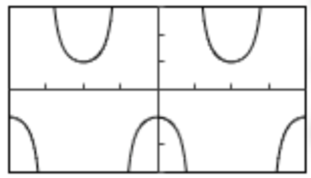
$$\cancel{-\frac{144}{169}}$$

$$\sqrt{\sin^2\alpha}=\sqrt{\frac{25}{169}}$$

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha = \frac{5}{13}\cdot\frac{7}{8} + \frac{\sqrt{15}}{8}\cdot\frac{-12}{13} = \frac{35-12\sqrt{15}}{104}$

To the right, half of an identity and the graph of this half are given. Use the graph to make a conjecture as to what the right side of the identity should be. Then prove your conjecture. The viewing window is $[-2\pi, 2\pi, \pi/2]$ by $[-3, 3, 1]$.

$$\frac{\cos(2x)}{\cos x} - \frac{\sin(2x)}{\sin x} = ?$$



$$\frac{\sin x \cdot (2\cos^2 x - 1)}{\sin x \cdot \cos x} - \frac{2\sin x\cos x \cdot \cos x}{\sin x \cdot \cos x}$$

$$\frac{\cancel{2\sin x\cos^2 x} - \sin x - \cancel{2\sin x\cos^2 x}}{\sin x\cos x} = \frac{-\cancel{\sin x}}{\cancel{\sin x}\cos x}$$

$$-\frac{1}{\cos x} = -\sec x$$